# EFFECT OF TESTING METHOD ON THE FATIGUE BEHAVIOR OF ASPHALT MIXTURES WITH HIGH RECLAIMED ASPHALT PAVEMENT (RAP) CONTENT

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# ABSTRACT

Instability in fuel prices and environmental concerns have encouraged the pavement industry to use high percentages of reclaimed asphalt pavement (RAP) in production of asphalt mixtures. Physical changes due to the addition of high RAP content, however, pose a challenge in mix design and, hence, affect the performance of resulting asphalt mixtures. To understand the effects of RAP content on asphalt mixture's fatigue behavior., asphalt mixes with various RAP content were tested using two different fatigue tests, conventional beam fatigue and push-pull (tension-compression) fatigue tests. The test is conducted under cyclic pure tensile and compressive stress states using gyratory compacted cores as the ones used complex modulus test. The test utilizes continuum damage theories, which might potentially be an appropriate test method especially for mixtures with high RAP content. The data from push-pull data has been analyzed using viscoelastic continuum damage (VECD) theory. The beam fatigue data, on the other hand, has been analyzed using conventional fatigue curves as well the plateau value (PV) concept. A comparison has been made between the number of cycles to failure obtained using both test data. The study concluded that the push-pull test is a promising method for testing mixes with RAP due to larger cross-sectional area and the ease of prepare specimens, using gyratory compacted specimens.

Keywords: Reclaimed asphalt pavement (RAP), Recycling, Beam fatigue test, Push-Pull fatigue test

# **1. INTRODUCTION**

In the U.S., asphalt pavement recycling has been practiced for four decades; but its significance has been recently highlighted again due to escalated fuel prices and growing interest in building sustainable pavement systems. Sustainable practices such as the use of reclaimed asphalt pavements (RAP) and warm mix asphalt (WMA) technology in pavement construction is being implemented globally. However, sustainable development can only be achieved when constructed pavements utilizing these materials can adequately withstand intended traffic and thermal loads. Therefore, the major challenges are that the prepared asphalt with recycled materials achieves the desired performance and design volumetrics while addressing the environmental concerns. The short-term and long-term performances and field volumetrics of the asphalt are inextricably linked; insufficiently compacted mixes in the field are more prone to develop segregation, cracking, and other distresses. Therefore, a comprehensive and thorough performance evaluation of mixes with RAP becomes critical to understand the contribution of such mixes towards building more sustainable pavements. Currently, federal and state departments of transportation in the U.S. are interested in increasing the amount of RAP in asphalt mixes to 50%. Physical changes due to the addition of high RAP percentages can, however, pose challenging mix design issues and significantly affect the performance of resulting asphalt. For example, voids in mineral aggregates (VMA) can play an important role in the performance of flexible pavements. Different researchers reported contradictory VMA trends with the addition of RAP. A decreasing trend of VMA has been observed with adding RAP [1, 2, 3, & 4]. On the other hand, Daniel and Lachance [5] and Hajj et al. [6] observed that the VMA of the asphalt increased with the addition of RAP. Increasing stiffness due to aged RAP bitumen is also another concern with the use of RAP, which might lead to premature cracking and reduced fatigue life.

There have been numerous research studies evaluating the various performance indicators of asphalt concrete (AC) with RAP using laboratory testing. It is believed that aged bitumen from RAP makes the resulting mix stiffer which, at one side, increases the moduli [5, 7] of the mix and its resistance against permanent deformation [8]. On the other hand, mixes with RAP can also become more prone to early thermal cracking [9, 10]. Regarding fatigue life of the asphalt with RAP, tests conducted in the NCHRP 9-12 study confirmed that asphalt with a RAP content greater than 20% has a lower fatigue life than the virgin AC [11]. In another study [12], the researcher demonstrated that the binder course asphalt containing 30% RAP outperformed the mix with no RAP when subjected to fatigue loading in indirect tensile test (IDT) setup. Shu et al. [13] observed that incorporating RAP increased the fatigue life of asphalt using the 50% stiffness reduction as a failure criterion. Whereas when the plateau values were used, RAP addition would cause input energy to turn into damage. Hence, that resulted in a shorter fatigue life.

Push-Pull (compression-tension) fatigue test assess fatigue the characteristics of asphalt using viscoelastic continuum damage theories. This test could reveal significant information about the potential impact of RAP on mixes. The test is conducted under cyclic pure tensile and compressive stress states using gyratory compacted cores, similar to the ones used for complex modulus test. Uniform tensile and compression stresses can be generated on a relatively larger surface area than classical beam fatigue tests. This might be particularly useful for mixes with high percentages of RAP. This test becomes more relevant when used for large aggregate asphalt avoiding local aggregate effects.

This paper aims at determining the effects of RAP on the fatigue characteristics of asphalt with RAP. This paper is part of a comprehensive study focused on evaluating the feasibility of incorporating high percentages of RAP (up to 50%) in asphalt [14]. Two fatigue tests (beam fatigue test and push-pull fatigue test) were considered and the performance of the asphalts was evaluated utilizing various damage criteria.

#### 2. MATERIAL AND MIX DESIGNS

Materials from two administrative districts (District 1 and District 5 in Illinois) of Illinois department of Transportation (IDOT) were used in this study. This paper discusses results from District 5 materials only. The material was collected from Open Road Paving at Urbana, IL. The source of the virgin aggregate is Vulcan Co. located at Kankakee, IL. Open Road Paving also provided two gradations of 19 mm nominal maximum aggregate size (NMAS) RAP, i.e. plus 9.5 mm and minus 9.5 mm. PG 64-22 was used as the base bitumen for designing the asphalt and two softer bitumen grades, i.e. PG 58-22 and PG 58-28 were used to evaluate the effect of bitumen bumping on the fatigue performance of the Asphalt with RAP. A control mix design with 0% RAP and three mix designs with 30%, 40%, and 50% RAP were developed in accordance with the IDOT specifications and utilizing Bailey method [15] of aggregate packing. Achieving consistent and acceptable volumetrics for asphalt mix design with high amount of RAP is extremely challenging due to their stockpile variability and relatively unknown aggregate characteristics. Bailey method and stringent mixture design and specimen preparation protocols proved to be extremely helpful in meeting the mix design volumetric requirements (VMA, VFA, etc.). The aggregate stockpile percentages and mix volumetrics are presented in Table 1. It is important to

note that a good consistency in VMA has been achieved for all mix designs. This ensures that the difference in performance of the four asphalts is not impacted by volumetric changes.

Volumetrics (%)	Control	30% RAP	40% RAP	50% RAP
Total Bitumen Content	5.2	5.2	5.2	5.2
Air Voids	4.0	4.0	4.0	4.0
VMA	13.8	13.8	13.6	13.5
VFA	71.0	71.0	70.8	70.4

Table 1: Volumetrics of asphalt mix designs

#### **3. BEAM FATIGUE TEST**

To determine the fatigue life of the prepared asphalts, a four point beam fatigue test was conducted. The test was run at 20 °C at strain levels of 1000, 800, 700, 500, 400, and 300 microstrains. A total of 60 beams were tested for the four asphalts. The failure criterion used in the study is the traditional 50% reduction in initial stiffness, whereas, the initial stiffness is the stiffness at 50<sup>th</sup> load cycle. Equation 1 shows a typical relationship between tensile strain at the bottom of the beam ( $\epsilon_o$ ) and number of load applications to crack appearance in the beam ( $N_f$ ).

$$N_f = K_1 \left(\frac{1}{\varepsilon_0}\right)^{K_2} \tag{1}$$

where  $K_1$  and  $K_2$  are the intercept and slope of a fatigue curve, respectively, and are dependent on composition and properties of AC. The higher the absolute value of  $K_2$ , the better the fatigue behavior of the mix is. Figure 1 shows the fatigue curves for the four mix designs evaluated in this study. The values of flexural moduli ( $E_f$ ), coefficient  $K_2$ , and dynamic moduli ( $E^*$ ) measured at 21 °C and 10 Hz are presented in Table 2. Modulus characteristics of these mixes were determined and presented elsewhere [14].

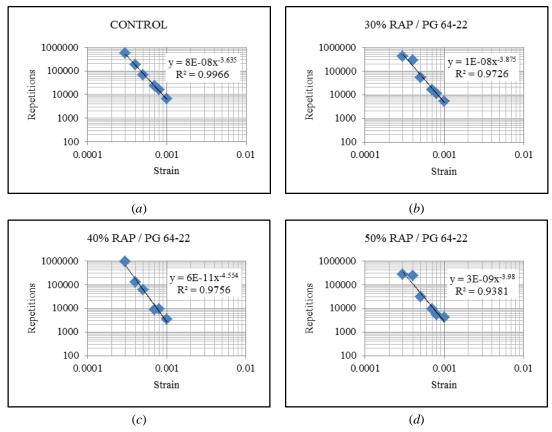


Figure 1: Fatigue curves for mixes with RAP at: (a) 0%; (b) 30%; (c) 40%; (d) 50%.

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	<b>K</b> <sub>2</sub>	E* (MPa)	(E <sub>f</sub> /E*)		
3314	3.64	7477	0.4		
4327	3.88	11390	0.4		
3579	3.80	9549	0.4		
3322	3.31	7222	0.5		
4864	4.55	11579	0.4		
4158	4.42	9410	0.4		
3695	4.24	8964	0.4		
5089	3.98	9903	0.5		
4175	4.78	8929	0.5		
4224	4.50	10071	0.4		
	$\begin{array}{c} \mathbf{E_{f}^{1}} \\ \textbf{(MPa)} \\ 3314 \\ 4327 \\ 3579 \\ 3322 \\ 4864 \\ 4158 \\ 3695 \\ 5089 \\ 4175 \end{array}$	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $		

 Table 2: Fatigue beam test results for all asphalts

 $^{1}E_{f}$  is initial flexural stiffness.

For the base bitumen PG 64-22,  $K_2$  values increased as RAP increased up to 40%; there is a slight decrease in the  $K_2$  values for AC with 50% RAP though. All the asphalts with RAP showed significant improvement in the fatigue behavior when softer bitumen, PG 58-22 and PG 58-28, was used. The  $E_f$  values increased as recycled material increased which was expected due to the relatively stiff RAP bitumen. For single bump PG 58-22 bitumen, the behavior of asphalts with 30% and 40% RAP remained approximately the same as the one with PG 64-22 bitumen. The asphalt with 50% RAP; however, showed significant (about 20%) improvement;  $K_2$  value increased from 3.98 to 4.78. The double bump bitumen PG 58-28 showed a reduction in fatigue life relative to the mixes using single bump bitumen. The  $K_2$  value for asphalt with 30% RAP is below the lower limit (3.5) of typical Illinois mixtures [16]. The double bumping effect is not that pronounced for asphalt mixtures with 40% and 50% RAP; there is still a reduction in K2 values compared to the single bumping results. It is important to note that bitumen bumping proved to be very effective in restoring the flexural modulus to that of control mix values.

Carpenter et al. [17, 18, and 19] proposed the concept of energy dissipation for determining the fatigue life of asphalts. A ratio of change in dissipated energy between two consecutive loading cycles divided by the dissipated energy of first cycle is termed as ratio of dissipated energy change (RDEC). Plateau value (PV), an almost constant value of RDEC, describes a period where there is a constant percentage of input energy dissipated due to damage accumulation in the specimen [20]. The damage in the specimen can be realized as microcrack evolution due to applied loads. Eventually these microcracks coalesce to form a macrocrack identifying the failure of the specimen, which can be easily recognized in RDEC plots. Herein, the PV concept (equation 2) was used to characterize the fatigue behavior of asphalt with RAP.

$$PV = 0.4428 N f_{50}^{-1.1102}$$
(2)

For a strain-controlled test, the lower the PV, the longer the fatigue life for a specific asphalt [19]. It can be speculated that plateau regime in the curve (as shown in Figure 2) indicates constant rate of energy dissipation due to microcrack evolution. As the rate increases in a strain controlled environment, the specimens can potentially reach failure prematurely.

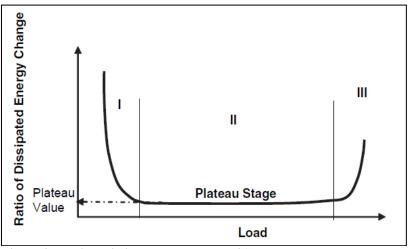


Figure 2: Typical dissipated energy plot [21]

Asphalt with higher RAP percentage exhibited an increase in the PV value which indicates reduction in their expected fatigue life (Figure 3). Although, this prediction contradicts the aforementioned observations using the traditional strain versus  $N_f$  approach; however, it is more intuitive to expect worse fatigue behavior from stiffer asphalts. The positive effect of bitumen bumping is also apparent in Figure 3 as shown with the reduction in PV for mixes PG 64-22 as compared those prepared with PG 58-28 and PG 58-22.

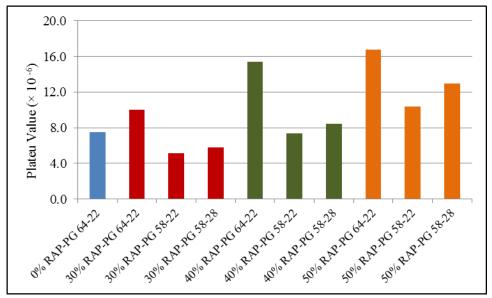


Figure 3: Plateau values for all mix designs

# 4. PUSH-PULL FATIGUE TEST

The cyclic push-pull (compression-tension) test was conducted in strain controlled testing mode. The effects of various percentages of RAP in asphalts on their fatigue properties were evaluated. The test was conducted at 10 Hz and two strain levels of 300 microstrains and 200 microstrains at 20 °C and 15 °C, respectively. It is important to note that during the test, the actuator strain levels were controlled instead of the three axial extensometers mounted on the specimen. Hence, a machine compliance factor was used to achieve target strains on the specimens. A fingerprint test was performed prior to the fatigue testing to determine the machine compliance factor and dynamic modulus ratio (DMR). DMR is the specimen variability compensation parameter and it usually has a value between 0.9 and 1.1 [22]. DMR is ratio between average representative dynamic modulus of the sample and dynamic modulus obtained in the fingerprint test. Figure 4 shows the experimental setup used for push-pull fatigue tests.



Figure 4: Push-pull test setup

According to the viscoelastic continuum damage theories, damage in asphalt is defined by a damage parameter (S) and pseudostiffness (C). The relationship between S and C was determined and expressed as damage characteristic curve using PP-VECD v0.1 software [23]. The test was terminated when the specimen reached failure criterion of 50% reduction in pseudostiffness (C), which is equivalent to 50% reduction in the initial stiffness defined by dynamic modulus |E\*|. The damage curve in Figure 5 illustrates stiffness reduction with increasing damage in the specimens. According to the continuum damage theories, damage can be interpreted as evolution of voids and/or microcracks as loading progresses. As damage increases, the specimen's load carrying capacity decreases; hence, the stiffness. A decrease in performance with an increase in the RAP content is evident in this figure with an exception of asphalt with 50% RAP. Figure 5 is plotted by fitting an exponential curve to C versus S data.

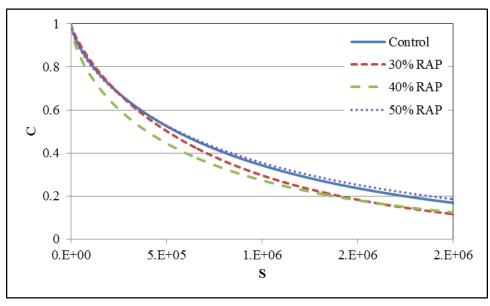


Figure 5: Pseudostiffness (C) vs. damage parameter (S) for all asphalts

The cycles to failure  $N_f$  were predicted by performing simulations at one frequency, different temperatures, and multiple microstrain levels using the PP-VECD v0.1 software. The following discrete formulation was used to calculate the  $N_f$  [24].

$$N_f = \sum_{S=1}^{S_f} \left[ -\frac{\varepsilon_0^2 |E^*|_{LVE}^2}{2} \frac{dC}{dS} \Big|_{at S} \right]^{-\alpha} f \Delta S_S \tag{3}$$

where  $|E^*|_{LVE}$  is the linear viscoelastic (undamaged) dynamic modulus, f is frequency, and  $S_f$  is the value of the damage parameter when C is equal to the pre-selected failure threshold. Figure 6 shows the fatigue curves for all the mix designs at 20 °C. A simple power curve was fitted on the VECD fatigue data in order to compare the K<sub>2</sub> values obtained from VECD fatigue data to that from the fatigue beam data. Table 3 shows the K<sub>1</sub> and K<sub>2</sub> values for the asphalt mix designs. Apart from asphalt with 50% RAP, K<sub>2</sub> values showed a decreasing trend with an increase in RAP content, suggesting a reduction in the fatigue behavior of the asphalts. These numbers are conflicting with the K<sub>2</sub> values obtained through the beam fatigue test. However, similar findings were observed when PV value method was used to interpret beam fatigue results. The improvement in fatigue behavior of 50% RAP mixes might be attributed to some experimental errors or variability in recycled materials. Additional tests will be conducted for the mixes with 50% RAP to recheck the data and better explain this behavior.

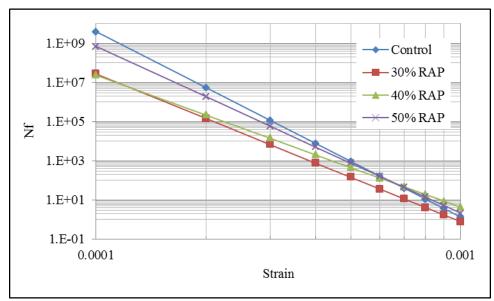


Figure 6: Fatigue curves obtained using N<sub>f</sub> by VECD method

Mix Type	$\mathbf{K}_{1}$	$\mathbf{K}_2$	
Control	5.00E-29	9.479	
30% RAP	1.00E-23	7.57	
40% RAP	2.00E-20	6.769	
50% RAP	6.00E-26	8.524	

 Table 2: Fatigue beam test results for all asphalts

#### 5. SUMMARY AND CONCLUSION

The use of recycled products in pavements is one approach to build environmental friendly pavements. However, the performance of asphalts with the addition of recycled products can be compromised. Laboratory mix preparation has also been a challenge; especially those with high RAP contents due to variability in RAP stockpiles. This study evaluated the fatigue performance of asphalt mixes with RAP. The fatigue behavior of the asphalts was studied using classical beam fatigue and push-pull fatigue tests. In case of beam fatigue test, two approaches to predict the fatigue behavior of the asphalts were compared i.e. conventional fatigue curve approach and PV approach. It was observed that the two methods gave contradictory results; the fatigue curve approach showed an improvement in fatigue behavior with RAP addition; whereas, the PV method showed that the addition of RAP might potentially reduce the asphalt fatigue life.

The damage curves (C versus S curve) obtained from the push-pull fatigue test suggested reduction in the performance of asphalts with high RAP content. The number of cycles to failure  $N_f$  determined using VECD approach also predicted inferior performance in asphalts with increasing RAP content which is in agreement with the results obtained by the beam fatigue test when PV method was employed to analyze the data. The push-pull test is a promising method for testing mixes with RAP due to larger cross-sectional area and the ease of specimen preparation, using gyratory compacted specimens.

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