DEVELOPMENT OF COMPUTATIONAL METHODS TO ESTIMATE PHASE ANGLE OF ASPHALT MIXTURES

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ABSTRACT
The recent practice in paradigm shifts in pavement design philosophy such as mechanistic-empirical pavement design has led to the development of several laboratory testing techniques in order to characterize asphalt pavement materials' responses such as viscoelastic properties. One of such important viscoelastic property of asphalt mixtures is phase angle, and its calculations from the outputs of the different frequency sweep tests is usually time consuming and error prone. The main purpose of this research study was to develop computational method(s) that can accurately estimate phase angle of asphalt mixtures for the data obtained from mechanical frequency-sweep tests such as stiffness modulus, dynamic modulus, shear modulus, etc. The efforts included collection of raw data from the various frequency sweep tests, and then estimate the phase angle values manually. To reduce the intensive manual work required and automatically estimate phase angle of an asphalt mix, two simple yet robust algorithms were developed using powerful mathematical functions and subsequently incorporated into computer programs. The first algorithm employed a curve-fitting approach upon each pulse of stress and strain signals of the frequency sweep test results with a polynomial function; the phase angle was calculated by locating the peak stress and strain of the fitted function. The second method involved transforming the stress and strain signals from time domain data to frequency domain using fast Fourier transform; the phase angle between the two signals was computed from the then transformed data. Both the algorithms were employed to calculate the phase angles of a few standard asphalt mixtures. A theoretical comparison revealed that the phase angle values obtained from the two developed methods were very close to each other's estimations but significantly different from the manually estimated phase angle values for the asphalt samples under consideration. This study dealt with only a limited amount of data. So, numerical comparisons were alone made to check for magnitudes and trends of the data manipulation. No statistical analyses were conducted to check for rationality and robustness of the data in comparison to the manual computation. At this time, the authors believe that Fourier transform method is much more powerful than the curve-fitting methodology owing to its capability to identify the test frequency and a corresponding phase angle value. Furthermore, in the curve-fitting method, one must input the corresponding raw data of loads and deformations for each frequency, which will then be used to fit curves to predict peaks and subsequently phase angle. Future work will encompass a comprehensive analysis, both at theoretical and stochastic levels to discern which method is best suited for adoption of automatic estimation of phase angles.

Key words: Asphalt mix, Phase angle, Frequency sweep, Curve-fitting, Fourier transform, Algorithm

1. INTRODUCTION
The response of bituminous layers in pavement structures subjected to varying traffic load and environmental conditions has led to the development of several laboratory testing procedures, mainly used to define and characterize different mechanical properties such as stiffness (or dynamic) modulus and phase angle. These mechanical parameters are considered most important to achieve performance-based design of bituminous layers.

Phase angle is one of the most important viscoelastic properties of asphalt concrete (AC) mixtures used to acquire quantitative information concerning the viscous and elastic nature of the mixtures. Also, this property has been well-utilized to characterize the pavement noise behaviour of the various asphalt materials (Biligiri, 2008; Biligiri *et al*, 2010). Recently, phase angle was also used as an important quantity in estimating the dissipated energy during dynamic load testing which subsequently was utilized for estimating the fatigue resistance of the asphalt mixtures based on energy method (Hamed, 2010).

A score of frequency sweep tests have been performed at VTI – Swedish National Road and Transport Research Institute, and other research institutes across the world to obtain stiffness (dynamic) modulus *(E*)* and phase angle (ϕ) values for the different asphalt mixtures. In the process of experimentation, huge amount of raw data is generated in the form of strains and stresses, which need to be manipulated to obtain the actual values of stiffness (dynamic) modulus and phase angles. Admittedly, calculations of E^* is not very difficult owing to its prime relation with stress and strain. However, estimation of ϕ is usually cumbersome and error prone due to manual calculations and the intricacy of its estimation based on the various inherent estimators that are correlated with strains in the time (or frequency) domain. To reduce the intensive manual work required to obtain ϕ of an asphalt mix from the outputs of dynamic frequency-sweep test, a simple yet a robust program is needed that can automatically calculate the accurate ϕ values using raw data. This paper presents an analytical methodology (algorithm) that was developed for the estimation of ϕ (and simultaneously E^*) based on fundamental mathematical functions. As part of the study, a simple computer program was also developed, whose theoretical principles will also be illustrated in the paper.

2. OBJECTIVE AND SCOPE OF THE WORK

The main purpose of this research study was to develop a computational method that can accurately estimate phase angle ϕ of asphalt mixtures for the data obtained from mechanical tests such as stiffness modulus, dynamic modulus, shear modulus etc.

The scope of the work included:

- Collect raw data from frequency sweep tests such as stiffness modulus, dynamic modulus and dynamic shear modulus tests.
- Estimate the phase angles values using manual computational method.
- Develop methodologies / algorithms to calculate the phase angle values based on simple mathematical functions.

3. THEORETICAL BACKGROUND

As mentioned previously, phase angle, ϕ is an important viscoelastic property characteristic of asphalt mixtures. ϕ is obtained as an output of frequency sweep testing simultaneously with stiffness property, such as complex or shear modulus, E^* (or sometimes G^*). Frequency sweep E^* test, for instance, was recommended as a simple performance test to complement the mixture design process under the National Cooperative Highway Research Program (NCHRP) Project 9-19 of the United States (NCHRP 465, 2002). The E^* is also an important input property in the mechanistic pavement design programs, such as Mechanical Empirical Pavement design Guide (MEPDG) for the design of new and rehabilitated pavement structures in the United States (NCHRP 1-37A, 2004).

Frequency sweep or sweep-frequency testing is used to quickly determine the broadband (range of frequencies) deformation or mechanical responses of asphalt mixtures that otherwise would require a number of separate tests for each frequency. Various frequency sweeps may be used to characterize asphalt mixtures' material responses during a mechanical test. Sweep-frequency techniques are applicable for dynamic modulus tests or dynamic shear modulus tests which are mainly utilized to characterize the permanent deformation and fatigue cracking performance behaviour of viscoelastic materials such as asphalt.

The tests are usually performed at several test temperatures in such a way that each specimen is tested in an order of increasing test temperature, and for each temperature the specimens are tested in an order of decreasing test frequency. This temperature-frequency sequence is carried out to cause minimum damage to the specimen before the next sequential test.

The outputs of frequency sweep tests are usually presented in the form of a master curve (for *E** and/or ϕ) which is a plot of the material property in question (usually *E** or ϕ) versus frequency (or time) at a standard temperature. Thus *E** or ϕ at any combination of test temperature and loading frequency can be obtained from the master curve.

The following sections provide a brief documentation of the commonly used frequency sweep tests, which are currently utilized by the pavement community around the world to obtain viscoelastic properties of the asphalt mixtures.

3.1 Dynamic modulus *E** test

*E** tests are conducted on unconfined cylindrical specimens having a height to diameter ratio of 1.5 and uses a uniaxially applied sinusoidal load (AASHTO TP62-07, 2006). For linear viscoelastic materials such as asphalt mixtures, the stress–strain relationship under a continuous sinusoidal loading is defined by a complex number called the complex modulus *E** (Pellinen, 2001; NCHRP 465, 2002). Figure 1 presents such a relationship between the various components associated with the E* test (subjected to uniaxial loading on a cylindrical specimen), such as stress (σ), strain (ε), time (t), phase angle (ϕ), and frequency (ω). Figure 2 represents an actual test setup for *E** test.

The dynamic modulus testing program involves testing at five different temperatures (-10, 4.4, 21.1, 37.8, and 54.4 ^{o}C) and six loading frequencies (25, 10, 5, 1, 0.5, and 0.1 Hz).

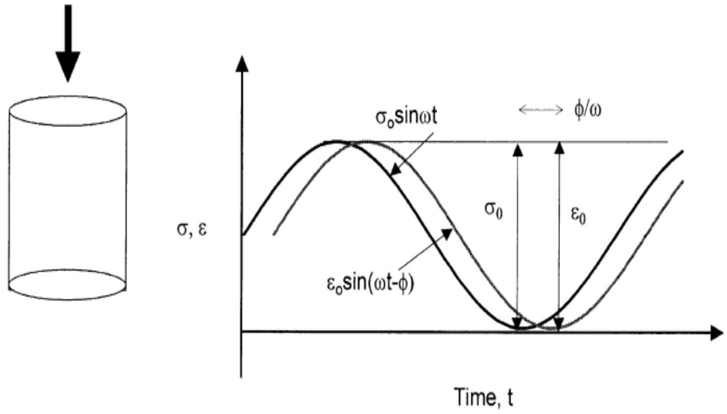

Figure 1: Representation of components in the dynamic modulus *E** test.

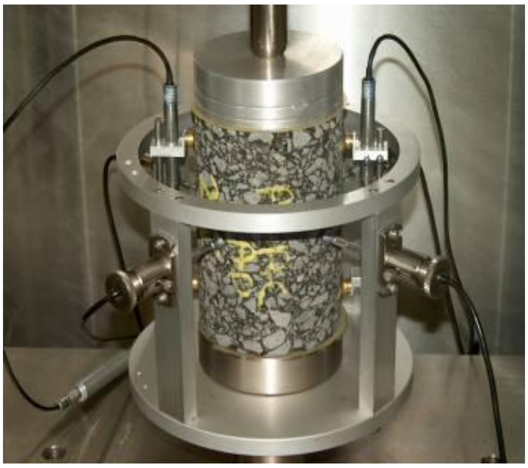

Figure 2: Actual *E** setup of an asphalt mix.

The dynamic modulus has a real component and an imaginary part that define the elastic and viscous behaviour of the linear viscoelastic material, respectively. These components are expressed as:

$$E^* = E' + iE'' \tag{1}$$

$$\left|E^*\right| = \frac{\sigma_o}{\varepsilon_o} \tag{2}$$

$$\begin{aligned} E' &= \left|E^*\right| \cos(\phi) \\ E'' &= \left|E^*\right| \sin(\phi) \end{aligned} \tag{3}$$

Where E^* is the complex modulus, σ_o, the peak stress and ε_o denotes the peak recoverable strain. *E'* and *E''* are usually referred to as storage (elastic) and loss modulus (viscous) components, respectively. The ratio of the energy lost to the energy stored in a cyclic deformation is referred to as loss tangent, *tan* (ϕ). The absolute value of the complex modulus, $|E^*|$, is defined as the dynamic complex modulus.
The phase angle, ϕ is simply the angle at which ε_o lags σ_o, and is an indicator of the viscous (or elastic) properties of the material under consideration. It is given by:

$$\phi = 360 \times f \times t_{lag} \tag{4}$$

Where t_{lag} is the average time lag (expressed in seconds) between stress and strain and *f* is the loading frequency (Hz). For a pure elastic material, $\phi = 0°$; and for a pure viscous material $\phi = 90°$.

3.2 Stiffness modulus test/Cyclic indirect tensile test

The stiffness modulus test or the cyclic indirect tensile test is considered as a simple and cost effective non-destructive laboratory test method for measuring the stiffness modulus of bituminous mixtures (SS-FAS 454, 1998; Kim *et al*, 2004). The test consists of applying a certain number of cyclic loading along the vertical plane of a specimen to achieve peak horizontal strain. The procedure is repeated by rotating the specimen through 90° and applying a second loading. The testing program includes testing at four different temperatures (-10, 0, 15, and 30 ^{o}C), and eight loading frequencies (20, 10, 5, 2, 1, 0.5, 0.2, and 0.1 Hz). Stiffness modulus / cyclic indirect tensile actual test setup is shown in Figure 3.

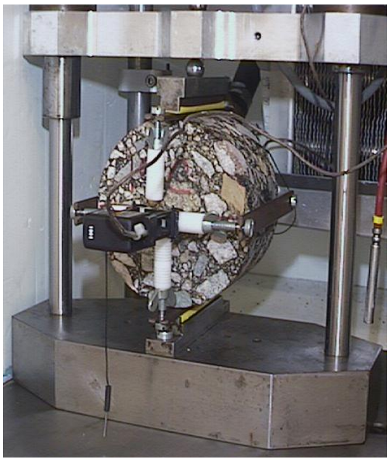

Figure 3: Actual cyclic indirect tensile test setup.

3.3 Dynamic shear modulus test

The dynamic shear modulus test is based on the method and the equipment developed at VTI. In this method, the specimen is glued between two steel plates with epoxy glue. One of the plates can be exposed to sinusoidal or repetitive loading over a range of frequencies (Said *et al*, 2011). The test is performed on cylindrical sample of 150 mm in diameter and the thickness of the specimen is less than ¼ of the diameter. Further information on the shear test can be found in (Said, 2004).

The dynamic shear testing procedure involves testing at five different temperatures, namely -7, 5, 20, 35 and 50 ^{o}C and 8 loading frequencies: 16, 8, 4, 2, 1, 0.5, 0.1, and 0.05 Hz. Figure 4 shows the shear box apparatus, which is used for dynamic shear modulus testing.

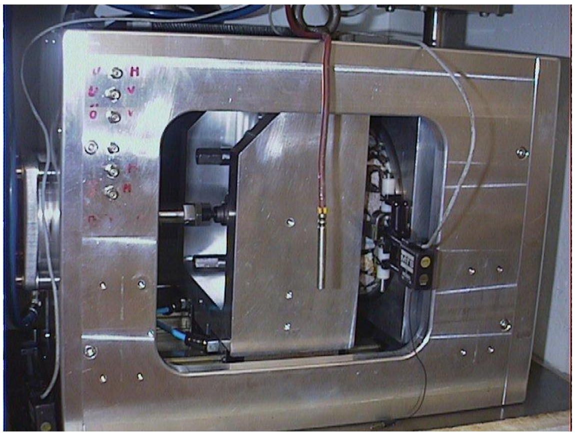

Figure 4: Actual dynamic shear modulus test setup used at VTI.

4. CALCULATION OF PHASE ANGLE, ϕ

Generally, ϕ obtained from frequency sweep tests are computed manually by locating the peak values of strains for their corresponding time delays using raw data. Manual computational procedure will not be illustrated in this paper since mostly always the choice of the peak strains is based on engineering judgment and the skill of the estimator. Thus, to avoid any sort of computation of erroneous ϕ results through the future, the authors deemed it important to develop a simple yet robust program to estimate ϕ using mathematical functions. Not only that, use of high-quality estimators (such as ϕ) will also enhance the importance of ϕ for further utilization as one of the major asphalt pavement materials' responses, which is often ignored by many across the world.

This section will detail the two different algorithms developed to estimate ϕ automatically using simple but powerful mathematical functions.

4.1 Phase Angle by Polynomial Curve-Fitting Method

In this method, polynomial functions, *s(t)* and *d(t)*, were used to approximate the load and deformation cycles respectively. Subsequently the time at which the maximum stress and maximum strain occur is obtained using elementary calculus. The phase lags or delays from all cycles are averaged to estimate the ϕ for each frequency. Mathematically, the location of the peak stress and strain can be calculated from:

$$\left.\frac{d(d(t))}{dt}\right|_{t=t_{\max,strain}} = 0 \quad (5)$$

$$\left.\frac{d(s(t))}{dt}\right|_{t=t_{\max,stress}} = 0 \quad (6)$$

$$t_{lag} = t_{\max,strain} - t_{\max,stress}$$

Where *d(t)* and *s(t)* are polynomial functions approximating the strain and stress. $t_{max,stress}$ and $t_{max,strain}$ are time (sec) values corresponding to the peak stress and strain, respectively.

Figure 5 presents typical curve fits for load and deformation pulses for an asphalt sample under consideration.

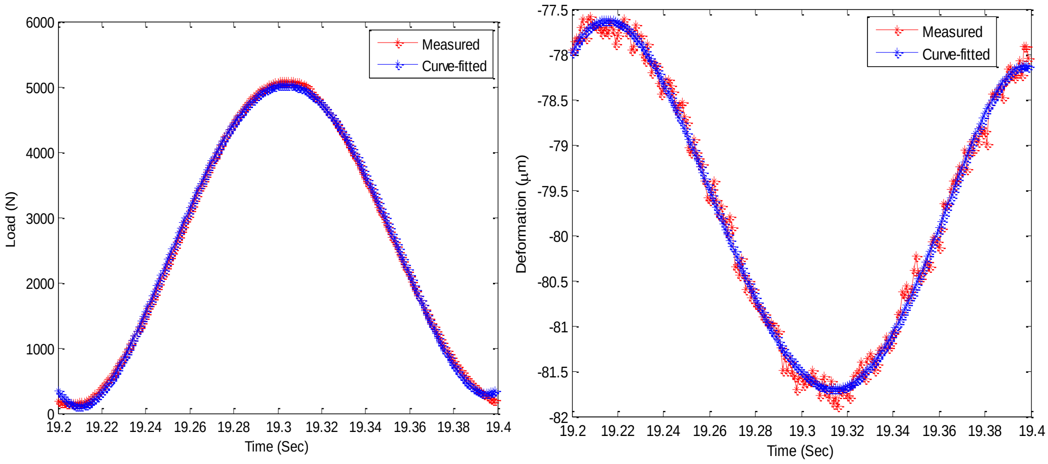

Figure 5: Curve-fitting of the measured load and deformation cycles versus time, approximated using 5th degree polynomial function.

The algorithm to perform the above procedure starts by preparing the data, and identifying the data for each frequency using the frequency tag in the data. Then, the pulse tag is used to select the data for each pulse in a given frequency. Then a curve is fitted to each pulse and the coefficient of determination, R^2 is calculated. If the R^2 value is not satisfactory, the portion of the data is truncated from both ends. Truncation of the data is performed mainly because the data required for phase angle calculation are the peak values and it is sufficient if good fit is attained in these territories. This process is iterated until the R^2 value is between 90 and 100%. The phase angle is then calculated from the locations of the peak values of strains and stresses. An algorithm presenting the procedure is as summarized in Figure 6.

4.2 Phase angle using Fourier Transform

The Fourier transform is a mathematical operation that decomposes a function into its constituent frequencies, known as its frequency spectrum. The composite waveform depends on time, and therefore is called the time domain representation. The frequency spectrum is a function of frequency and is called the frequency domain representation. Each value of the function is a complex number that contains information regarding the magnitude and phase of each frequency component. Mathematically, the Fourier transform of function *f(x)* is given by (Gilbert, 2007):

$$\hat{f}(\zeta) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i x \zeta} dx \tag{7}$$

Where $\hat{f}$ denotes the Fourier transform of the function and ζ is the frequency.

In relation to the Fourier transform, Cross Power Spectrum (CPS) of two signals, *g(t)* (stress signal) and *h(t)* (strain signal) is defined as Fourier transform of the cross correlation function of the two signals. The cross correlation, ρ_{gh}, of the *g(t)* and *h(t)* is given by:

$$\rho_{gh}(\tau) = \int_{-\infty}^{\infty} g(t) h(t+\tau) dt \tag{8}$$

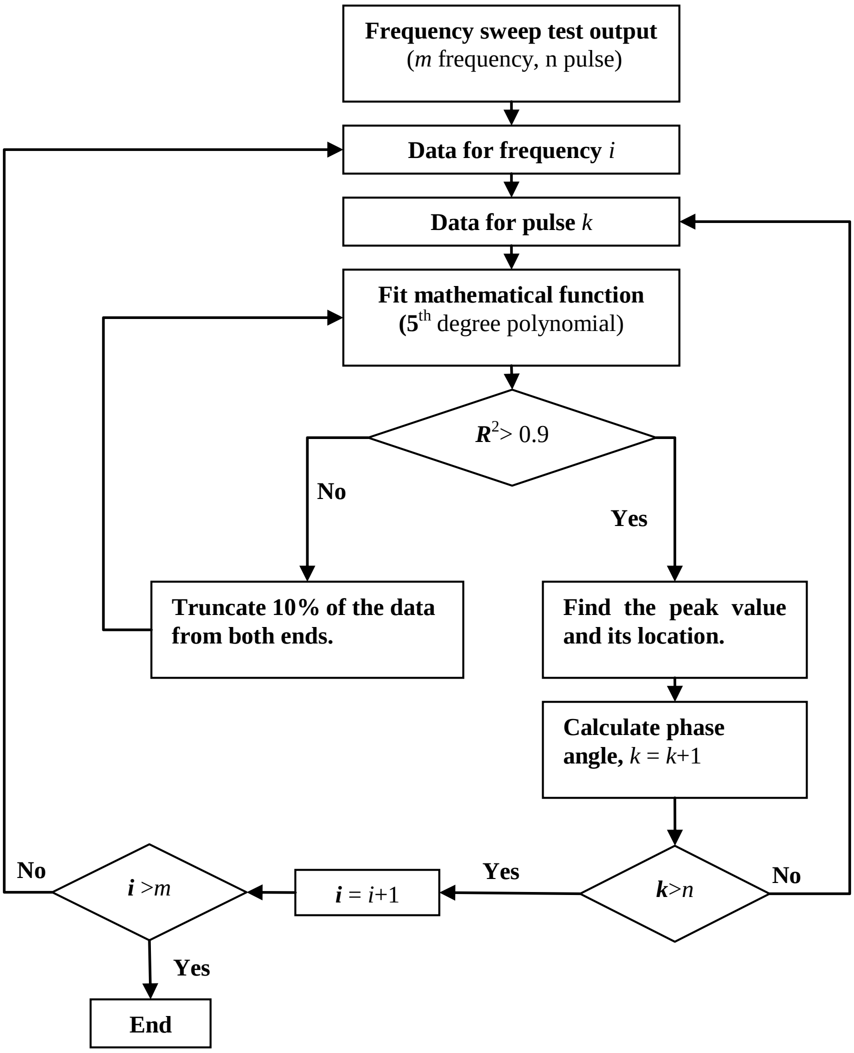

Figure 6: Flowchart representation of phase angle calculation using curve-fitting function.

The CPS of *g(t)* and *h(t)* is thus, cross correlation theorem using the properties of Fourier transform:

$$Y_{gh}(\zeta) = \int_{-\infty}^{\infty} \rho_{gh} e^{-2\pi i x \zeta} dx = \frac{\hat{g}(\zeta)\overline{\hat{h}(\zeta)}}{N^2} \tag{9}$$

where $\hat{g}(\zeta)$ and $\overline{\hat{h}(\zeta)}$ are the Fourier transform of *g(t)* and complex conjugate of the Fourier transform of *h(t)*, respectively, Y_{gh} is the CPS and N is the sample size of the data.

The CPS shows the strength of the signal (energy) as a function of frequency. In other words, it shows frequencies at which the signal is strong or weak. As CPS is a complex valued function, it contains both magnitude $|Y_{gh}(\zeta)|$ and ϕ between *g(t)* and *h(t)*.

Thus to obtain ϕ based on this method, the stress and strain signals are first transformed from time domain to frequency domain using Fourier transform and ϕ is calculated from the real and imaginary part of CPS of the then transformed data. Mathematically, ϕ is given by:

$$\phi = \tan^{-1}\left(\frac{imag(Y_{gh})}{real(Y_{gh})}\right) \tag{10}$$

As this approach is dependent on the trend in the deformation signal, which might exist due to a permanent deformation, a proper method should be incorporated to remove the trend before transforming it to a frequency domain. The process is summarized in Figure 7.

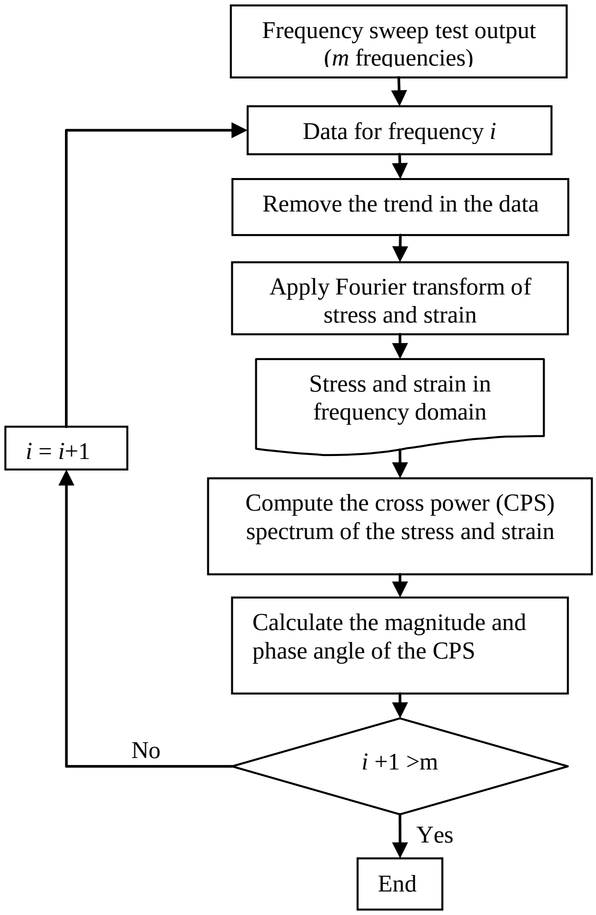

Figure 7: Flowchart representation of phase angle calculation using Fourier transform.

5. RESULTS AND ANALYSES

The two methods developed in this study to calculate phase angle of an asphalt mix were employed to estimate ϕ of an actual asphalt concrete mix tested at VTI using E^* test. Figures 8 through 10 present results of the estimated ϕ values of an asphalt concrete mixture (tested at 21 ^{o}C and six frequencies), based on the Fourier transform method.

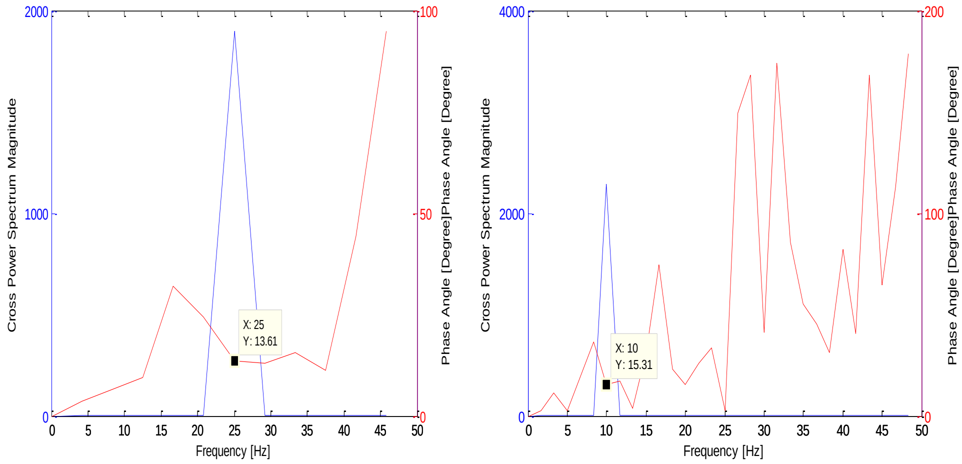

Figure 8: Phase angle estimations at 25 and 10 Hz, Fourier transform method.

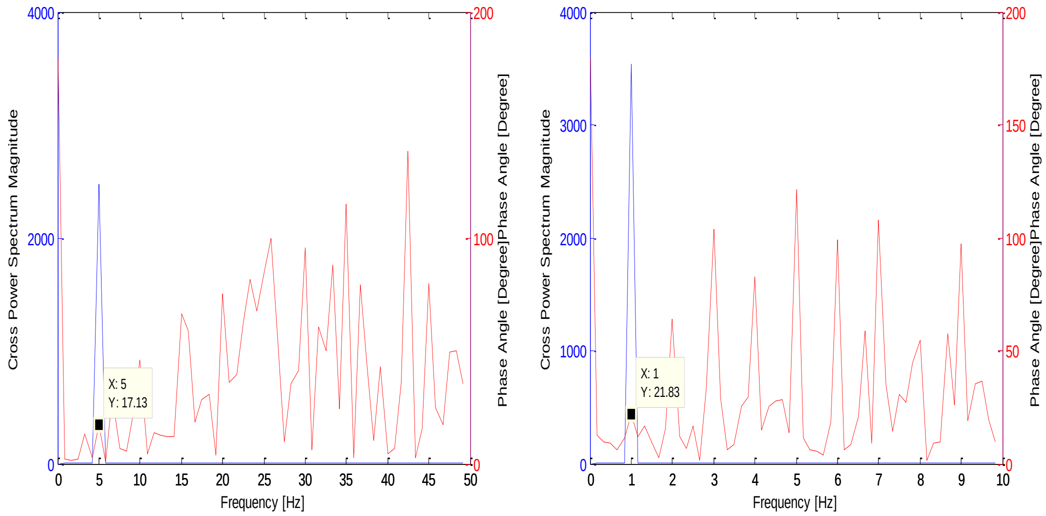

Figure 9: Phase angle estimations at 5 and 1 Hz, Fourier transform method.

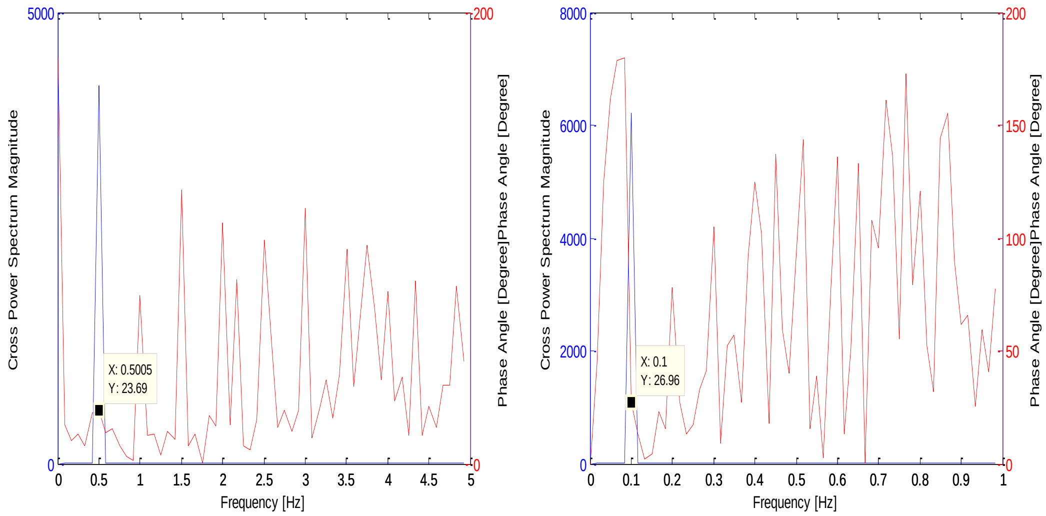

Figure 10: Phase angle estimations at 0.5 and 0.1 Hz, Fourier transform method.

As observed in the figures, Fourier transform method enabled to identify the test frequency and its corresponding ϕ. Consequently, to distinguish between the different test frequencies under consideration and their corresponding ϕ values, the frequencies contained in the stress and strain signals have been represented by blue lines and the ϕ estimated using Fourier transform method are shown using red lines.

Figure 5 previously presented illustrated a general representation of the curve fitting procedure for an asphalt sample. The figure also depicted how a polynomial curve (shown using blue lines) was fitted upon the actual raw data (represented by red lines) for both load and deformation pulses. For the lack of space in the document, plots of polynomial curve fitting for the various *E** test frequencies will not be shown in the paper. But, one must understand that the computer program developed by the authors is capable of estimating the ϕ from the peak values of the load versus time and deformation versus time domains for each frequency sweep using the polynomial curve fitting method. However, unlike the Fourier transform method, curve-fitting method does not identify test frequency automatically; instead, one must input the corresponding raw data of loads and deformations for each frequency, which will then be used to fit curves to predict peaks and subsequently ϕ. It is important to note that the same sample represented in the Figure 5 was used also to estimate ϕ by Fourier transform method as represented by Figures 8 through 10.

Figure 11 presents a plot of frequency and ϕ estimated using both curve-fitting and Fourier transform methods for an asphalt sample tested at 21 ^{o}C and six frequencies. Likewise, the developed ϕ prediction procedures were employed to estimate ϕ using the data obtained from two other frequency sweep tests, namely, stiffness modulus and dynamic shear modulus tests. Figures 12 and 13 show the estimated ϕ obtained from curve-fitting and Fourier transform methods, for stiffness modulus and dynamic shear modulus tests, respectively.

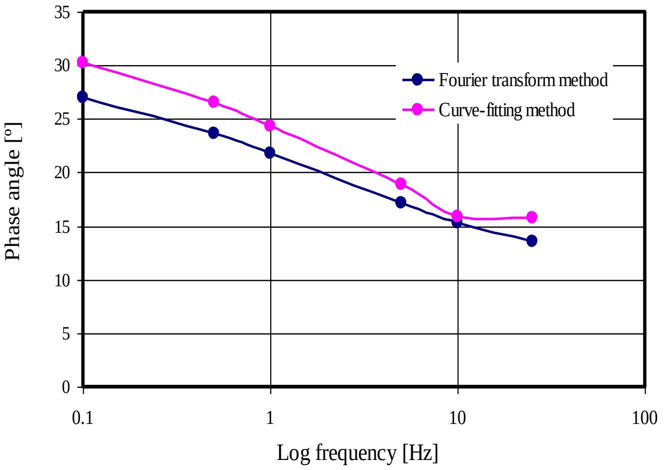

Figure 11: Computed phase angle from dynamic modulus E* test of an asphalt sample tested at 21 ^{o}C and six frequencies.

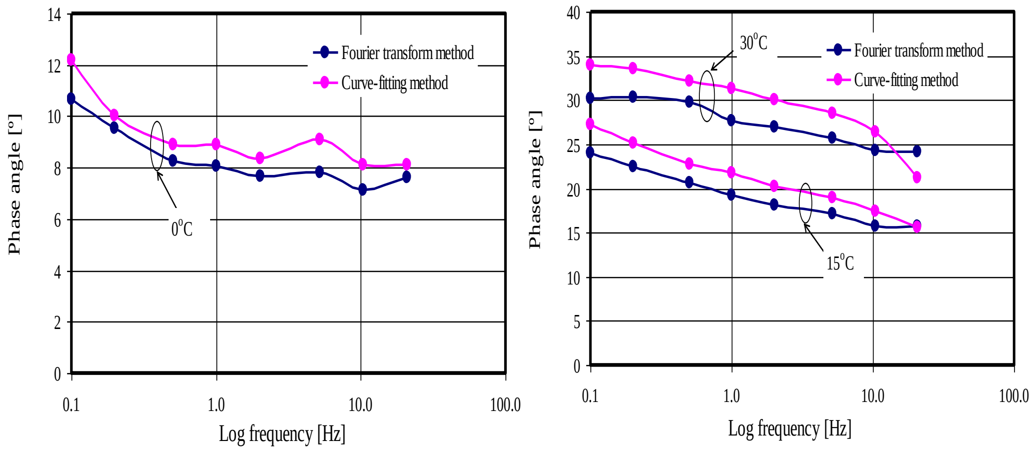

Figure 12: Computed phase angle from stiffness modulus test of an asphalt sample tested at 0, 15 and 30 ^{o}C, and eight frequencies.

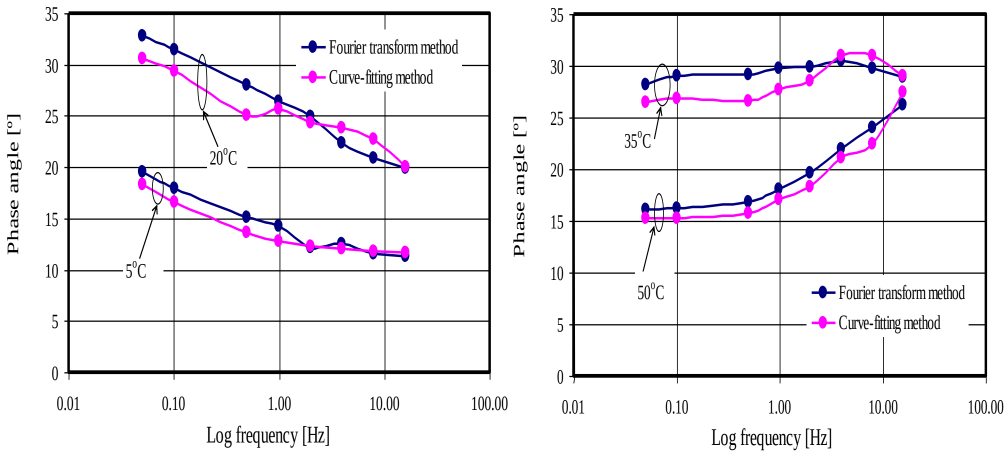

Figure 13: Computed phase angle from dynamic shear modulus test of an asphalt sample tested at 5, 20, 35 and 50 °C, and eight frequencies.

A numerical comparison revealed that the ϕ values obtained from the two developed methods were very close to each other's estimations but significantly different from the manually estimated ϕ values for the asphalt samples under consideration. However, the phase angle values obtained using the curve-fitting method are slightly higher than the values obtained using the Fourier transform method for both dynamic modulus E* and stiffness modulus tests. Furthermore, phase angle values obtained using the both procedures for dynamic shear modulus tests were of negligible difference just by comparing the magnitudes of the values.

Phase angle master curves were also constructed to comprehensively observe the trends of the ϕ values across a wide range of test temperatures and frequencies for dynamic shear modulus tests; ϕ obtained using curve-fitting and Fourier transform methods. Figure 14 presents the ϕ master curves for an asphalt sample for the data obtained from dynamic shear modulus test. The shapes of the phase angle master curves of the asphalt sample for the data obtained from the dynamic shear modulus test are similar to what is normally expected for such tests. However, a smoother master curve is obtained from the Fourier transform method than the curve-fitting method.

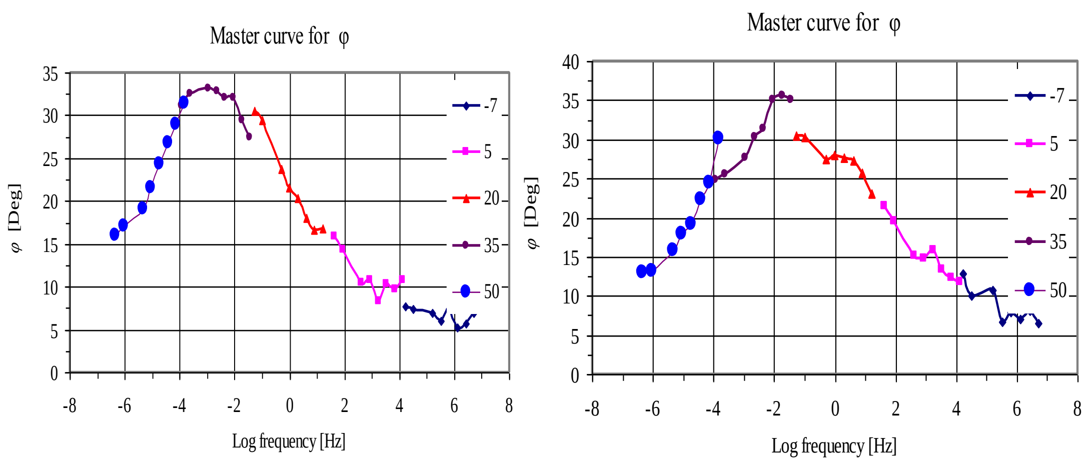

Figure 14: Phase angle master curve from dynamic shear modulus test of an asphalt sample (Left using the Fourier transform, right using the curve-fitting).

The discrepancies in the ϕ values of the two methods may be attributed to the inherent differences in the mathematical approaches adopted. Essentially, the curve-fitting approach applies fitting a curve locally, i.e., for each pulse of stress and strain signals; while, the Fourier transform method tries to fit the whole signal in a comprehensive sense, similar to the so-called “spectral methods” (Gilbert, 2007).

6. DISCUSSION

In this paper, two simple yet powerful user-friendly methods were developed using robust mathematical functions to calculate phase angles of asphalt mixtures from the different frequency sweep tests such as dynamic modulus test, cyclic indirect tensile test, and dynamic shear modulus tests. Both the methods were developed with an intention to avoid any errors that might occur during manual calculations and also to save a considerable amount of time. Basically, the methods were used to estimate a parameter incorporated with sinusoidal repeated loading. However, it is anticipated that the methods developed in the study could be possibly expanded and used in the estimation of components for other types of periodic loading under different domains of mechanical testing. Since this study dealt with only a limited amount of data, numerical comparisons were alone made to check for magnitudes and trends of the data manipulation. No statistical analyses were conducted to check for rationality and robustness of the data in comparison to the manual computation. Thus, the future work will encompass investigating the two developed methodologies for a larger amount of data. A comprehensive analysis, both at theoretical and stochastic levels would be performed to come to a conclusion as to which of the two algorithms would be accurate and rational.

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